Chs. 29/30: EM Waves, Reflection, Refraction
Tuesday November 15th

- **V. IMPORTANT: Final exam will be in HCB103/316**
  - **There will be assigned seating (TBA)**
- **Mini-exam 5 on Thursday (AC circuits and EM waves)**
- **55 unregistered *i*Clickers – any takers?**

- Finish Electromagnetic waves (Ch. 29)
  - Review: wave solutions and relations between quantities
  - Energy flux and intensity
- Reflection and Refraction (Ch. 30)
  - Wave reflection from an interface
  - Wave transmission through an interface (refraction)
  - Snell's law
  - Total Internal reflection
  - Dispersion

Reading: up to page 540 in the text book (Ch. 29/30)

Maxwell's equations

Table 29.2 Maxwell's Equations

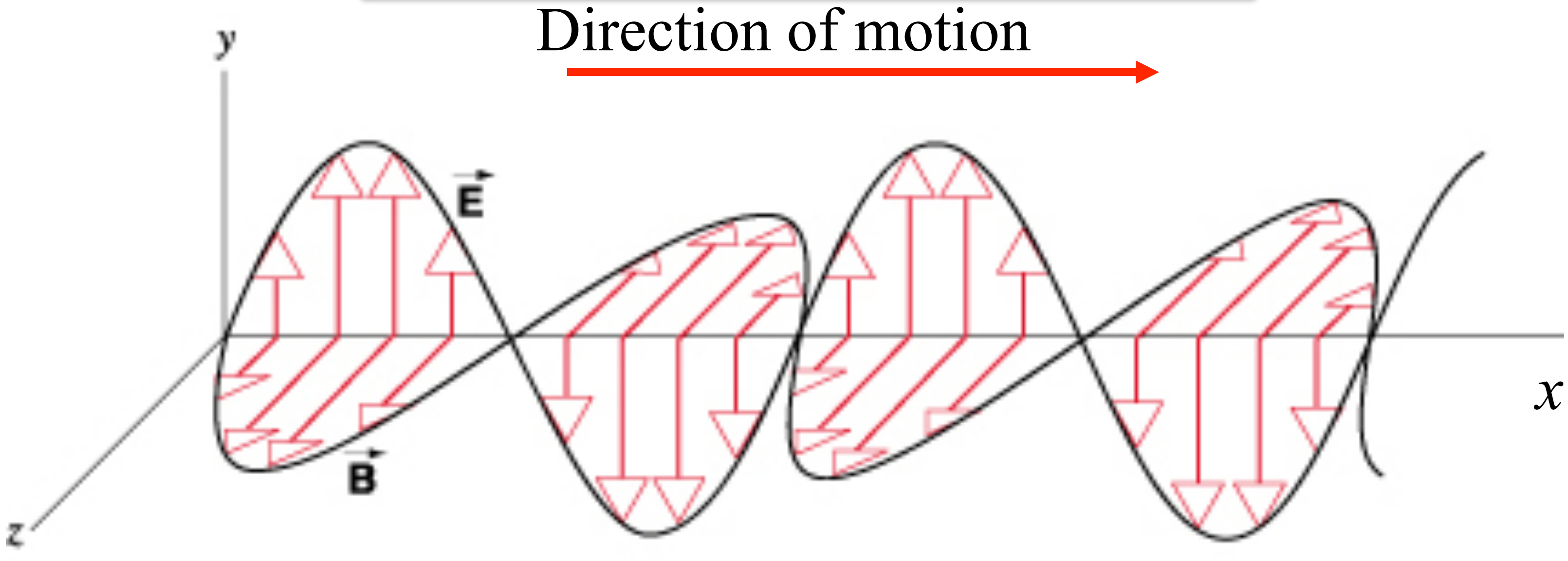
Law	**Mathematical Statement**	**What It Says**
Gauss for $\vec{E}$	$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$	How charges produce electric field; field lines begin and end on charges.
Gauss for $\vec{B}$	$\oint \vec{B} \cdot d\vec{A} = 0$	No magnetic charge; magnetic field lines don't begin or end.
Faraday	$\oint \vec{E} \cdot d\vec{r} = -\frac{d\Phi_B}{dt}$	Changing magnetic flux produces electric field.
Ampère	$\oint \vec{B} \cdot d\vec{r} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$	Electric current and changing electric flux produce magnetic field.

The main thing to note here is the symmetry in the last two equations: a time varying magnetic field produces an electric field; similarly, a time varying electric field produces a magnetic field.

Electromagnetic waves

- The *E* and *B* fields are still related via Ampère's and Faraday's laws.
- For a plane wave traveling in the x direction (see text):

$$\boxed{\begin{aligned}\vec{\boldsymbol{E}}\left(x,t\right) &= E_{\mathrm{p}}\sin\left(kx-\omega t\right)\hat{\boldsymbol{j}} \\ \vec{\boldsymbol{B}}\left(x,t\right) &= B_{\mathrm{p}}\sin\left(kx-\omega t\right)\hat{\boldsymbol{k}}\end{aligned}}$$



Electromagnetic waves

- Plugging these wave solutions into the wave equation:

$$\nabla^2 E_y = -k^2 E_y = \mu_o \varepsilon_o \frac{\partial^2 E_y}{\partial t^2} = -\omega^2 \mu_o \varepsilon_o E_y$$

$$\Rightarrow \boxed{\frac{\omega^2}{k^2} = c^2} = \frac{1}{\mu_o \varepsilon_o}, \qquad \text{or} \qquad \boxed{c = \sqrt{\frac{1}{\mu_o \varepsilon_o}}}$$

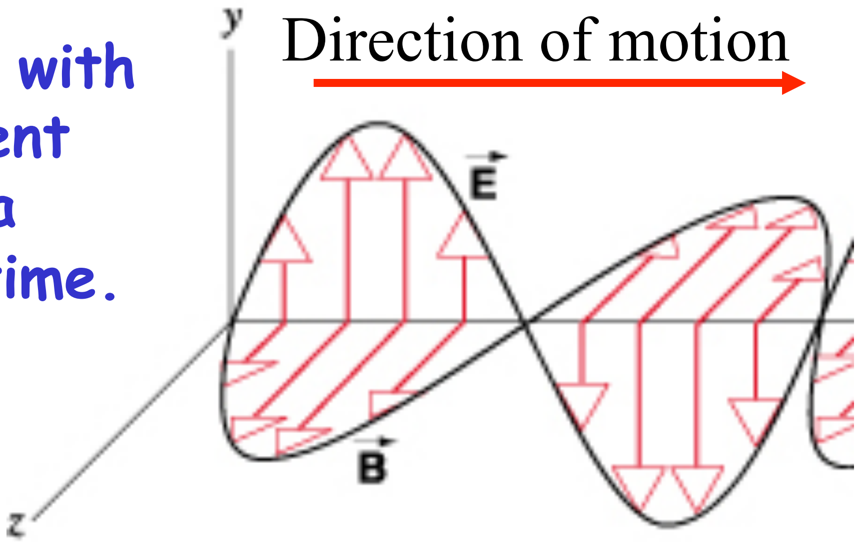
- Plugging these wave solutions into Faraday's law:

$$\frac{\partial E_y}{\partial x} = kE_p \cos\left(kx - \omega t\right) = -\frac{\partial B_z}{\partial t} = \omega B_p \cos\left(kx - \omega t\right)$$

$$\boxed{\Rightarrow \frac{E_p}{B_p} = \frac{\omega}{k} = c}$$

Poynting vector and light intensity

This is the energy 'flux' associated with the EM wave - like an 'energy current density' or energy crossing unit area perpendicular to the flow, per unit time.



$$\boxed{\vec{S} = \frac{1}{\mu_o}\vec{E}\times\vec{B}}$$

Right-hand rule

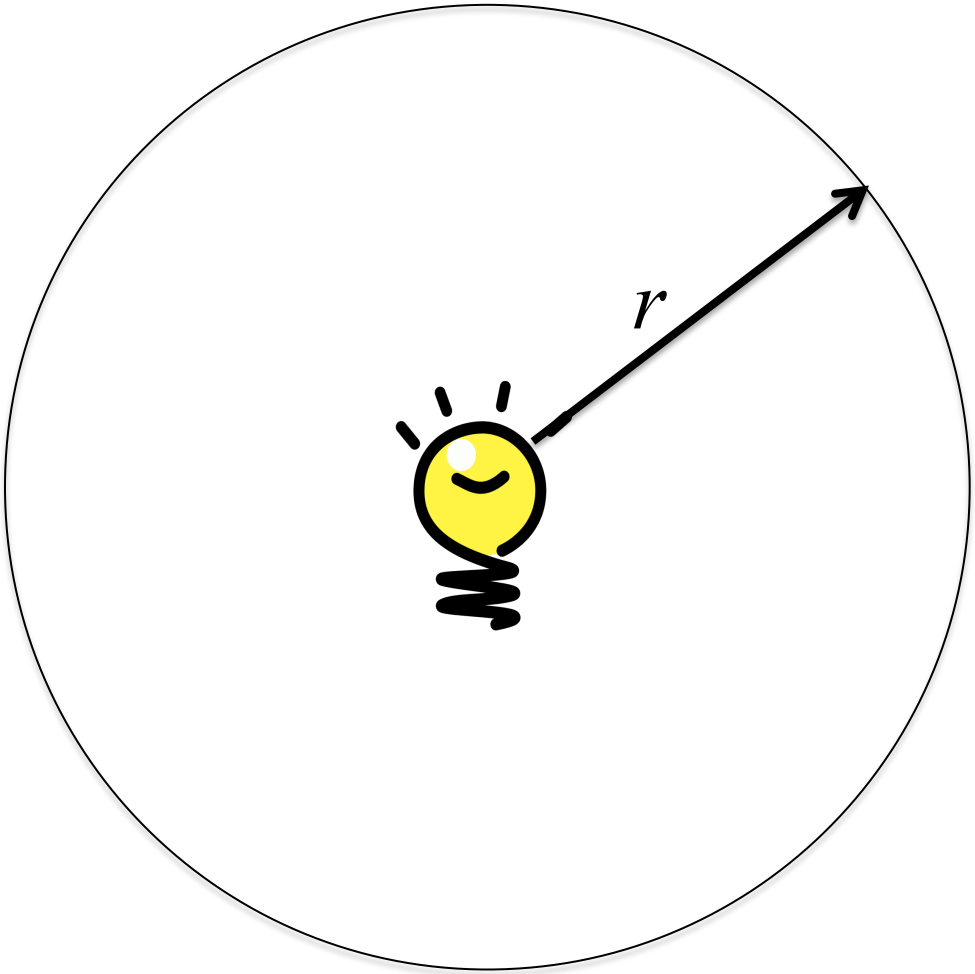
$$\boxed{S = \frac{1}{\mu_o}EB} = \frac{1}{\mu_o c}E^2 = \varepsilon_o c E^2 = \frac{c}{\mu_o}B^2$$

Intensity (average rate of energy incidence per unit area):

$$\boxed{I = S_{av}} = \langle S\rangle = \frac{1}{\mu_o c}E_p^2\left\langle \sin^2\left(kx-\omega t\right)\right\rangle = \boxed{\frac{1}{2\mu_o c}E_p^2}$$

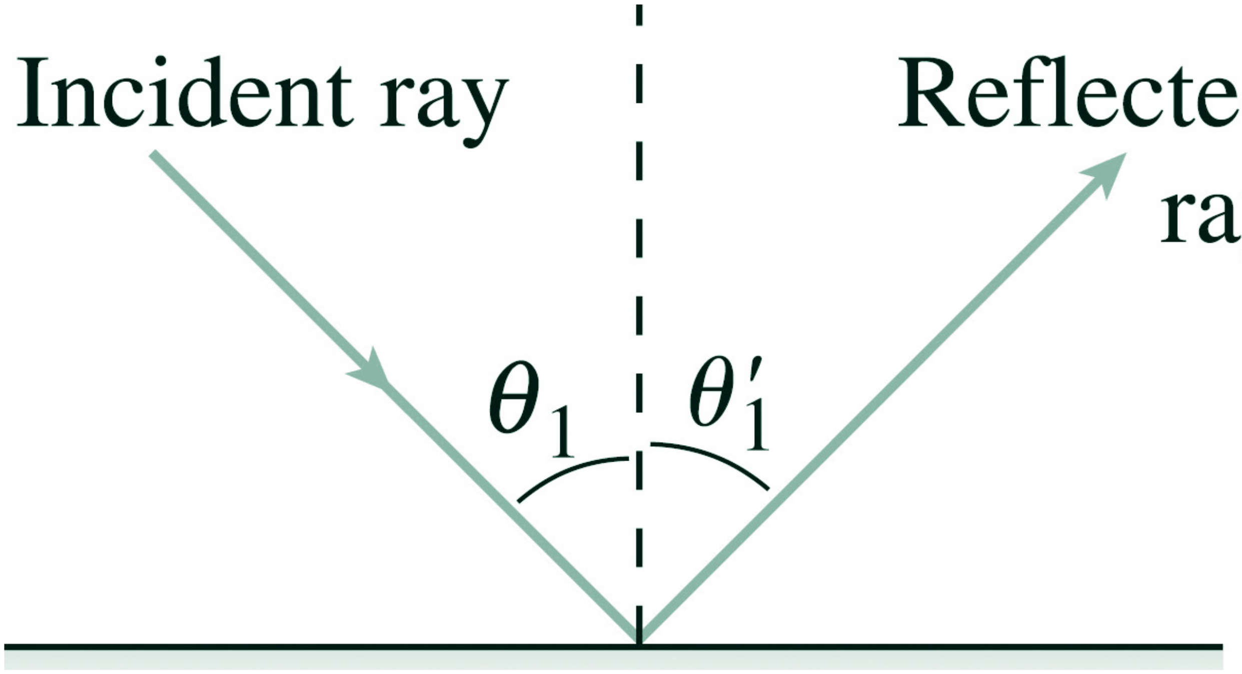
Intensity from a Point Source

Consider a light source that emits uniformly in all directions [note: no single oscillator could do this, but a large number of oscillators can, e.g., a light bulb.]



$$I = \langle S \rangle = \frac{P}{4\pi r^2}$$

Wave Reflection (Ch. 30)



$$\boxed{\theta_1' = \theta_1}$$

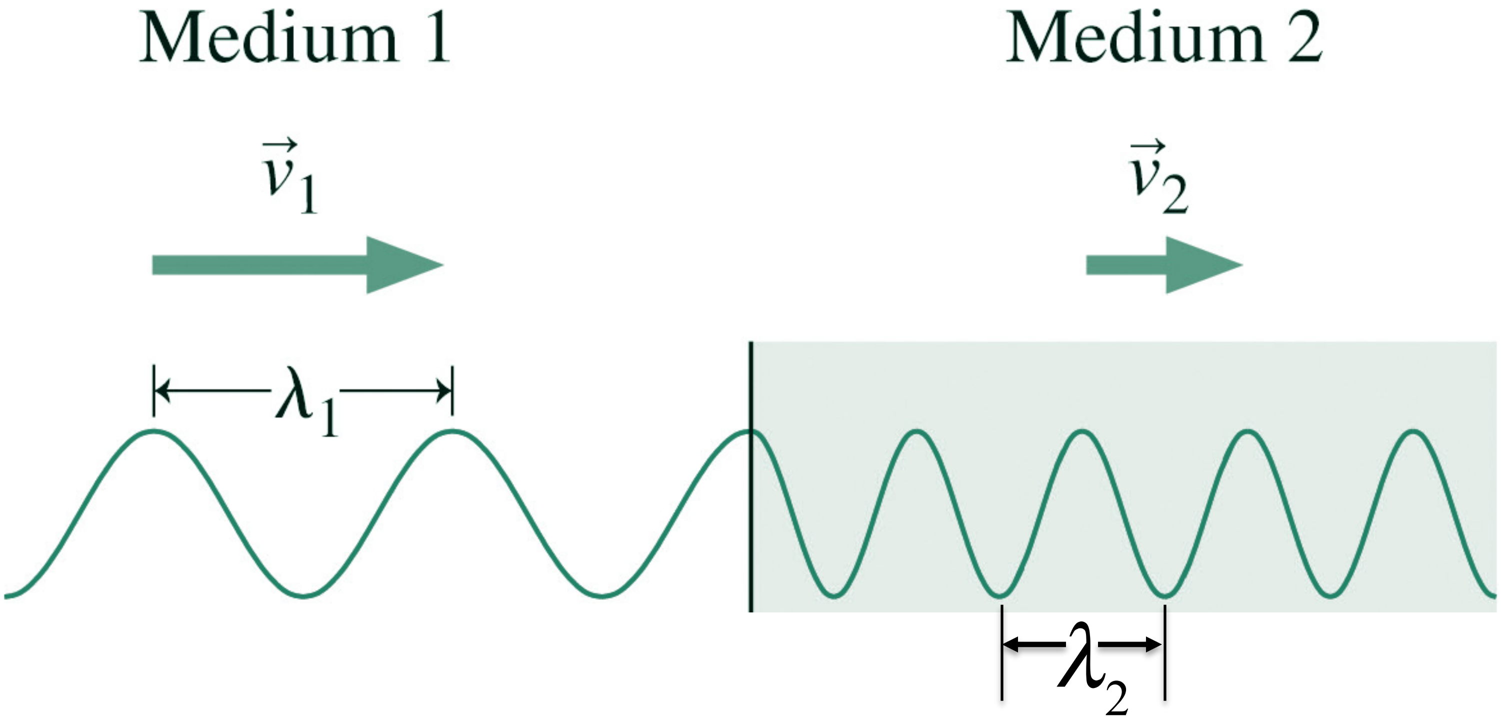
- There are a number of different ways to rationalize this, both in terms of the wave- and particle-like nature of light.
- The latter involves conservation of energy/momentum, i.e., just like a perfect elastic collision between a billiard board and the rail.

Refractive index

When a wave travels into a medium other than vacuum, the constants ε_o and μ_o are modified by their permeabilities κ_e and κ_m, thus the speed of the electromagnetic wave is given by:

$$v = \sqrt{\frac{1}{\kappa_e \kappa_m}} \sqrt{\frac{1}{\mu_o \varepsilon_o}} = c\sqrt{\frac{1}{\kappa_e \kappa_m}} = \frac{c}{n},$$

where $n = (\kappa_e \kappa_m)^{1/2}$ is called the refractive index of the material.



$$f_1 = f_2$$

$$\boxed{\frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}}$$

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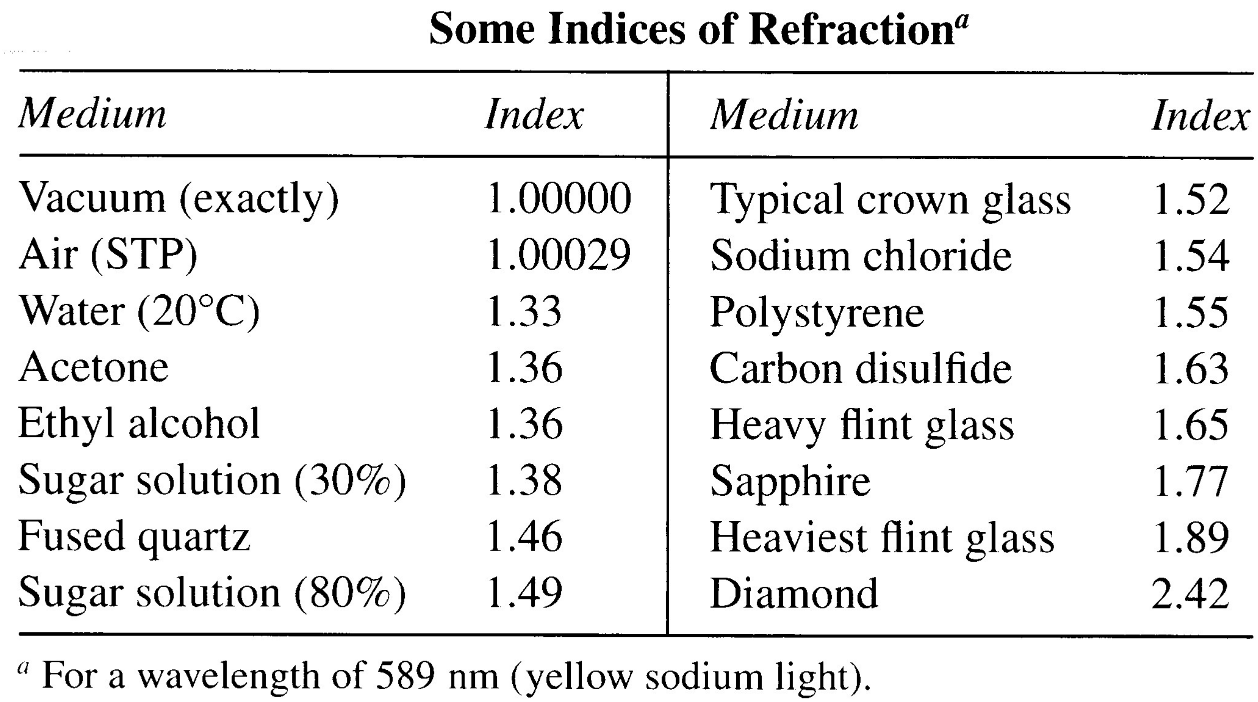
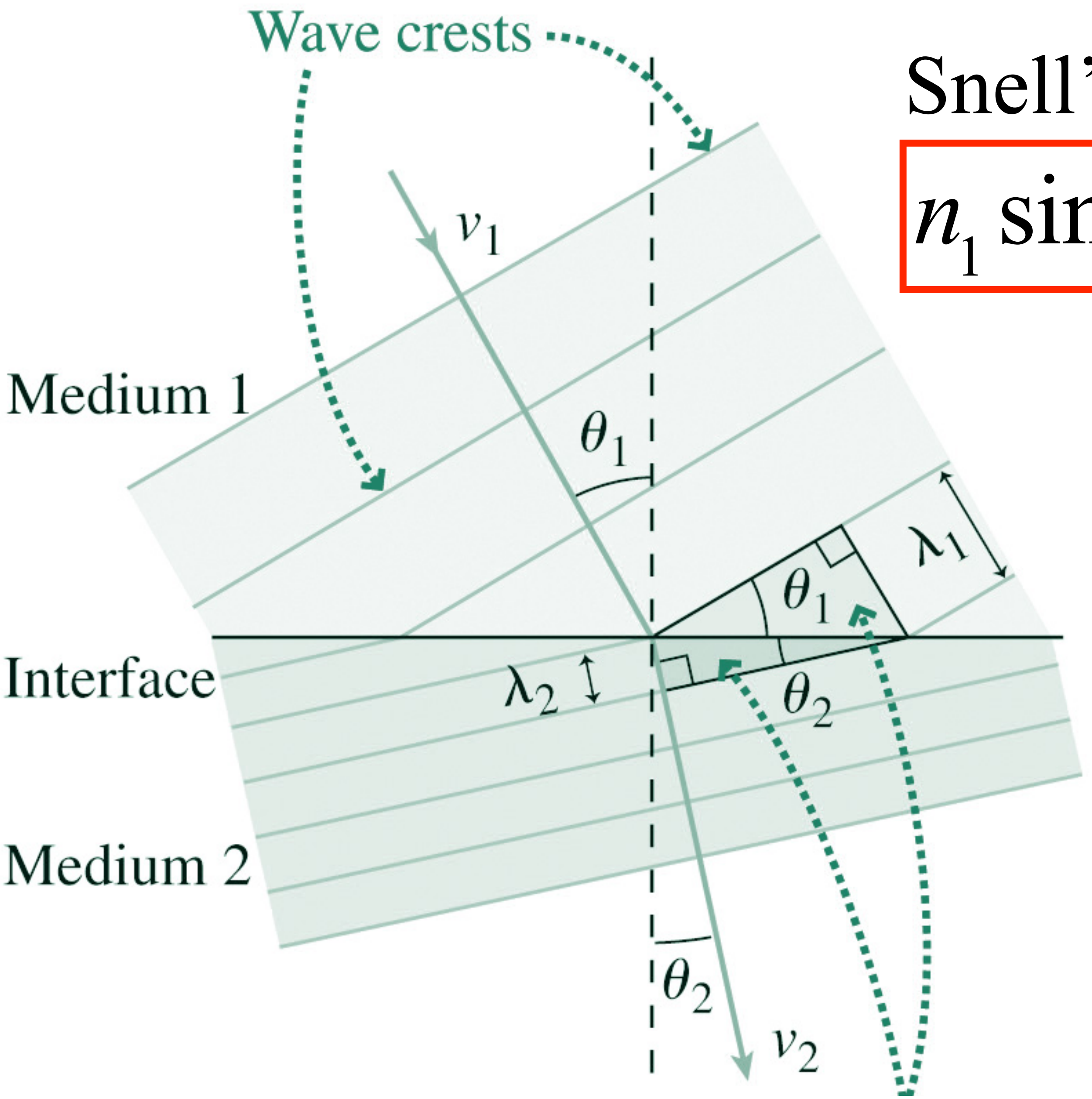


Some Indices of Refraction[a]

Medium	*Index*	*Medium*	*Index*
Vacuum (exactly)	1.00000	Typical crown glass	1.52
Air (STP)	1.00029	Sodium chloride	1.54
Water (20°C)	1.33	Polystyrene	1.55
Acetone	1.36	Carbon disulfide	1.63
Ethyl alcohol	1.36	Heavy flint glass	1.65
Sugar solution (30%)	1.38	Sapphire	1.77
Fused quartz	1.46	Heaviest flint glass	1.89
Sugar solution (80%)	1.49	Diamond	2.42

[a] For a wavelength of 589 nm (yellow sodium light).

Refraction and Snell's law



Snell's law:

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$